Optimization problems for energy markets' transport infrastructure

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ORM 2016
MOSCOW

Introduction

Markets of natural gas, oil, electricity and other resources play an important role in economies of many countries. An essential component of such markets is a transmission system. Consumers and producers are located at different nodes, and transmission capacities of the lines between the local markets are limited. By recent estimates, the transmission costs may exceed 50% of the electricity price for the industry consumers in Russia.This costs are also rather large for natural gas and oil markets.


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Владивосток
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По оценкам экспертов и компаний.
* проект уже окупился; ** проект заморожен; *** проект работает в убыток
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Introduction

The previous researches on such markets (Davidson et al., 2009; Hogan, 1998) consider primarily models with a fixed network structure. The recent paper (Daylova, Vasin, 2014) determines the optimal transmission capacity of one line for a two-node market, taking into account transmission losses and costs of transmission line construction. The present study aims to generalize these results for markets with several transmission lines.

Introduction

We consider the total welfare optimization problem with account of the production costs, consumers' utilities and the costs of transmission lines expansion. The optimal solution of this problem determines the total welfare value that can further be reallocated by means of special economic mechanisms letting one to obtain any Pareto-optimal outcome. The difficulty of the problem is that an expansion of any line requires valuable fixed costs. If the optimal set of expanded lines was known, the problem would be convex. However, the efficient search of this set requires special methods. In general, the problem is NP-hard (Guisewite, Pardalos, 1990). We distinguish several cases where the social welfare function is submodular or supermodular with respect to the set of expanded transmission lines. So the known excluding rules (Cherenin, 1962; Khachaturov, 1989) are applicable.

Formal model of the market

Let N denote the set of nodes and $L \subseteq N \times N$ be the set of edges. Every node $i \in N$ corresponds to a local perfectly competitive market. Demand function $D_i(p)$ and supply function $S_i(p)$ meet standard conditions. The consumption utility function: $U_i(q) = \int_0^q D^{-1}(v)\, dv$. $S_i(p) = Arg \max_v (pv - c_i(v))$, where $c_i(v)$ is the minimal production cost of volume v at node i. The total profit of producers at node i is $Pr_i(\overline{p}) = \int_0^{\overline{p}} S_i(p) dp$.

Formal model of the market

For any $(i, j) \in L$ the transmission line is characterized by the initial transmission capacity Q_{ij}^0, the unit transmission cost e_t^{ij}, the cost of the transmission capacity increment, including fixed cost e_f^{ij} and variable cost $e_v^{ij}(Q_{ij}, Q_{ij}^0)$. Let q_{ij} be the flow from the market i to market j. The total transmission costs for edge (i, j) are:

$$E_{ij}\left(q_{ij}\right) = \begin{cases} e_f^{ij} + e_v^{ij}\left(\left|q_{ij}\right|, Q_{ij}^0\right) + \ e_t^{ij}\left|q_{ij}\right|, & \text{if} \quad \left|q_{ij}\right| > Q_{ij}^0 \ , \\ e_t^{ij}\left|q_{ij}\right|, & \text{if} \ \left|q_{ij}\right| \leq \ Q_{ij}^0 \ . \end{cases} \tag{1}$$

The final transmission capacity Q_{ij} is $\left|q_{ij}\right|$, if $\left|q_{ij}\right| > Q_{ij}^0$, otherwise it is Q_{ij}^0. The cost of the line expansion is the overnight construction cost amortized over the life-time T_{ij} of the line using discount rate r: $e^{ij} = r\frac{OC_{ij}}{1-e^{rT_{ij}}}$; e_v^{ij} is a monotonous convex function of increment $(Q_{ij} - Q_{ij}^0)$.

The total social welfare

Denote $Z(i)$ the set of nodes connected with node i. Under fixed flows $\overrightarrow{q} = (q_{ij}, (i,j) \in L)$ and production volumes $\overrightarrow{v} = (v_i,\ i \in N)$, the consumption volumes $(\widehat{v}_i,\ i \in N)$ are $\widehat{v}_i = v_i\ +\ \sum_{j \in Z(i)} q_{ji},\ i \in N$. The total social welfare for the network market is

$$\overline{W}(\overrightarrow{q}, \overrightarrow{v}) = \sum_{i \in N} [U_i \left(v_i + \sum_{l \in Z(i)} q_{li} \right) - c_i(v_i)] - \sum_{(i,j) \in L, \quad i<j} E_{ij}(q_{ij}). \tag{2}$$

An alternative representation of this value is the total profit of all the agents in the market: producers, consumers and the transmission system.

The total social welfare

Indeed, under strategies $\overrightarrow{Q},\ \overrightarrow{v}$, the price $p_i(\overrightarrow{Q},\ \overrightarrow{v})$ at node i meets the balance equation: $D_i\left(p_i\right) = v_i + \sum_{j \in Z(i)} q_{ji},\ \ i \in N$. The producers' profit is $Pr_i = p_i v_i - C_i\left(v_i\right)$, the consumers' surplus is $CS_i = \int_{p_i}^{\infty} D_i(p)dp$, and the benefit of the transmission system is determined as $T\left(\overrightarrow{p}, \overrightarrow{Q}\right) = \sum_{(i,j)\in L,\ i<j} [\left(p_j\left(\overrightarrow{Q},\ \overrightarrow{v}\right) - p_i\left(\overrightarrow{Q}, \overrightarrow{v}\right)\right) q_{ij} - E_{ij}(Q_{ij})]$.
Then
$\overline{W}\left(\overrightarrow{Q}, \overrightarrow{v}\right) = \sum_{i\in N}(Pr_i(\overrightarrow{Q}, \overrightarrow{v}) + CS_i(\overrightarrow{Q}, \overrightarrow{v})) + T\left(\overrightarrow{p}, \overrightarrow{Q}\right)$.
The problem under consideration is

$$\max_{\overrightarrow{q}, \overrightarrow{v}} \overline{W}(\overrightarrow{q}, \overrightarrow{v}) \tag{3}$$

The total social welfare

Modification of the total welfare concept for the case where some final nodes are either exporting or importing the good. At an exporting node, the transmitted good is sold to foreign consumers. The total social welfare component is $W^i(q_{\sigma(i)i}) = q_{\sigma(i)i} \cdot D_i^{-1}(q_{\sigma(i)i})$, where $\sigma(i)$ is the preceding node for node i. At the importing node $W^i(p_i) = -q_{i\sigma(i)} S_i^{-1}(q_{i\sigma(i)}) = -S_i(p_i) \cdot p_i$. Similar modifications should be specified for "foreign" companies operating in the national market.

Theorem 1

Under any fixed flows of the good between the local markets $(q_{ij},\ (i,j) \in L)$ *, the optimal production volume at node* i *is* $v_i = S_i(\widetilde{p}_i)$*, where* $\widetilde{p}_i$ *meets equation* $\triangle S_i\left(\widetilde{p}_i\right) = \sum_{j \in Z(i)} q_{ij}$*,* $\triangle S_i\left(p_i\right) = S_i\left(p_i\right) - D_i\left(p_i\right)$ *denotes the supply-demand balance.*

Formal model of the market

For any $\overline{L} \subseteq L$, consider problem (3) with fixed set $\overline{L}$ of expanded lines. That is, $|q_{ij}| \leq \; Q^0_{ij}$ for $(i,j) \in L \setminus \overline{L}$, and the fixed costs are always included in E_{ij} for $(i,j) \in \overline{L}$.

Theorem 2

The latter problem is convex, and its solution $(\overrightarrow{q}, \overrightarrow{v})(\overline{L})$ meets FOCs which determine the competitive equilibrium of the corresponding network market.

Let $\widetilde{W}(\overline{L})$ denote the maximal welfare in the latter problem. Then problem (3) reduces to $\max_{\overline{L} \subseteq L} \widetilde{W}(\overline{L})$.

Formal model of the market

Consider problem (3) without construction costs and under constraint: $|q_{ij}| \leq\ Q_{ij}, (i,j) \in L$. Let $\widetilde{p}_i(\overrightarrow{Q}), i \in N$, denote the equilibrium prices corresponding to the solution of this problem:

$(v_i(\overrightarrow{Q}), i \in N)$ meet Th.1, $\forall (i,j) \in L$

$p_j(\overrightarrow{Q}) > p_i(\overrightarrow{Q}) + e_t^{ij} \Rightarrow q_{ij} = Q_{ij}$

$p_j(\overrightarrow{Q}) - p_i(\overrightarrow{Q}) < e_t^{ij} \Rightarrow q_{ij} = 0$

$\Delta S_i(p_i(\overrightarrow{Q})) = \sum_{j \in Z(i)} q_{ij}$

Definition 1

The model under consideration meets the flow structure invariance condition if, for any
$\overrightarrow{Q} \geqslant \overrightarrow{Q}^0, (i,j) \in L,\ sign(p_i(\overrightarrow{Q}) - p_j(\overrightarrow{Q})) = sign(p_i(\overrightarrow{Q}^0) - p_j(\overrightarrow{Q}^0))$.

Properties of the welfare function for multi-node network markets

Function $\widetilde{W}(\omega)$ defined for each subset $\omega \in \bar{L}$ of finite set $\bar{L}$, is *submodular* (respectively, *supermodular*) if for every $L', L'' \subseteq \bar{L}$ it holds that
$\widetilde{W}(L') + \widetilde{W}(L'') \geq \ (respectively, \leq)\ \widetilde{W}(L' \cup L'') + \widetilde{W}(L' \cap L'')$.
Let $\widetilde{W}$ be submodular.

1. If $\forall i \in L$ $\widetilde{W}(L) \geq \widetilde{W}(L \setminus \{i\})$, then $\max_{S \subseteq L} \widetilde{W}(S) = \widetilde{W}(L)$.
2. Let $\widetilde{W}(i) < \widetilde{W}(\emptyset)$. Then $i \notin S^* := Argmax_{S \subseteq L} \widetilde{W}(S)$.
3. For some S, i let $\widetilde{W}(S \cup \{i\}) \leq \widetilde{W}(S)$. Then $\forall R \supset S$ $\widetilde{W}(R \cup \{i\}) \leq \widetilde{W}(R)$.

The dual properties determine optimization algorithms for a supermodular function.

Properties of the welfare function for multi-node network markets

Consider an algorithm that permits to efficiently find the optimal set L^* for submodular function $W(L)$ in a typical case where the fixed costs are sufficiently large.

Step 1: First, consider separate lines and determine the set $\mathfrak{L}_1 = \{l : \quad \widetilde{W}(l) > \widetilde{W}(\emptyset)\}$ including such lines that it is more profitable to increase their capacities than to invest nothing in them.

Step $k = 2, 3, \ldots$**:** Assume by induction that we have determined the set $\mathfrak{L}_{k-1}$ of $(k-1)$-tuples $L_{k-1} = \{l_1, \ldots l_{k-1}\}$ such that $\widetilde{W}(L_{k-1}) > \widetilde{W}(\overline{L})$ for any subset of lines $\overline{L} \subset L_{k-1}$. Now, we determine the set $\mathfrak{L}_k$ of k-tuples L_k such that, for any $L_{k-1} \subset L_k$, the following conditions hold: $L_{k-1} \in \mathfrak{L}_{k-1}$ and $\widetilde{W}(L_{k-1}) < \widetilde{W}(L_k)$.

For every $\mathfrak{L}_k$ we consider the set M_k of maximal k-tuples $\overrightarrow{l}^k = (l_1, \ldots, l_k)$ such that for any l_{k+1} the tuple $(l_1, \ldots, l_{k+1}) \notin \mathfrak{L}_{k+1}$, and we choose the optimal $(\overrightarrow{l}^k)^* = \mathrm{argmax}_{\overrightarrow{l}^k \in M_k} W(\overrightarrow{Q}^*(\overrightarrow{l}^k))$. We find $k^* = \mathrm{argmax}_k \widetilde{W}((\overrightarrow{l}^k)^*)$.

Properties of the welfare function for multi-node network markets

The desirable properties of the welfare function closely relate to the flow structure invariance under any increment of transmission capacities.

Figure 1: Flow structures that determine (a) supermodular and (b) submodular welfare functions

In general, a chain-type market may include both structures as its components and meet none of the conditions of super- or submodularity.

Properties of the welfare function for multi-node network markets

Theorem 3

For a chain-type market with n nodes, let the initial prices $p_i(\overrightarrow{Q}^0)$, $i = 1, 2, \ldots, n$, monotonously decrease in i. Then, for any $\overrightarrow{Q} \geq \overrightarrow{Q^0}$, $p_i(\overrightarrow{Q}) \geq p_{i+1}(\overrightarrow{Q})$, $i = 1, 2, \ldots, n-1$, and the function $\widetilde{W}(\overline{L})$ is supermodular. The complexity of the search for the optimal set $\overline{L}^*$ under $\overrightarrow{Q^0} = 0$ does not exceed $\frac{(n-1)n}{2}$.

Properties of the welfare function for multi-node network markets

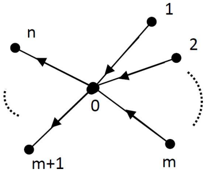

Figure 2: Star-type market

Consider a star-type market with $n+1$ nodes such that, under initial transmission capacity $\overrightarrow{Q^0}$, 0 is a transit node, $I_1 = \{1, 2, \ldots, m\}$ is a set of producing nodes, $I_2 = \{m+1, \ldots, n\}$ is a set of consuming nodes.

Properties of the welfare function for multi-node network markets

Theorem 4

For a star-type market, the flow directions invariance holds if and only if $\forall i \in I_1$ $p_i(\overrightarrow{Q^0}||Q^{\infty}_{I_1}) \geq p_0(\overrightarrow{Q^0}||Q^{\infty}_{I_1})$*, and* $\forall i \in I_2$ $p_i(\overrightarrow{Q^0}||Q^{\infty}_{I_2}) \leq p_0(\overrightarrow{Q^0}||Q^{\infty}_{I_2})$*, where* $Q^{\infty}_i = \infty$*,* $i \in I$*.*

Theorem 5

Consider a star-type market that meets the condition of flow directions invariance. The social welfare function $\widetilde{W}(L_1 \cup L_2)$*, where* $L_1 \subseteq I_1$*,* $L_2 \subseteq I_2$*, is submodular in* L_1 *under fixed set* L_2 *and is also submodular in* L_2 *under fixed set* L_1*. Moreover, for any* $L_1 \subseteq I_1$*,* $l \in I_1 \setminus L_1$*, the social welfare function increment* $\widetilde{W}(l \cup L_1, L_2) - \widetilde{W}(L_1, L_2)$ *monotonously increases in set* L_2*, and for any* $L_2 \subseteq I_2$*,* $l \in I_2 \setminus L_2$*, the social welfare function increment* $\widetilde{W}(L_1, l \cup L_2) - \widetilde{W}(L_1, L_2)$ *monotonously increases in set* L_1*.*

Properties of the welfare function for multi-node network markets

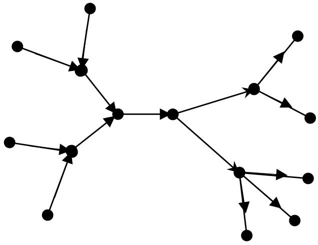

Figure 3: Tree-type market

Some generalizations

Definition 2

Edge $l \in L$ is called complementary (resp. competitive) to edge $q \in L$ if, for any $M \subseteq L \setminus [l, q]$, $W(M \cup (q, l)) - W(M \cup l) \geqslant (\text{resp.} \leqslant) W(M \cup q) - W(M)$ Let $M_1(q)$ denote the set of complementary and $M_2(l)$ the set of competitive edges for edge l. Obviously, $W(L_1 \cup L_2 \cup l) - W(L_1 \cup L_2) \uparrow L_1 \subseteq M_1(l)$ and $\downarrow L_2 \subseteq M_2(l)$

So algorithms similar to proposed for sub- and supermodular functions are applicable if for any $l \in L$ we can determine $M_1(l)$, $M_2(l)$ and $M_1(l) \cup M_2(l) = L \setminus l$

Complementary and competitive edges for chain-type markets

Consider *a* chain-type market. Let
$L_1 = \{(i, i+1) | p_{i+1}(\overrightarrow{Q}^0) > p_i(\overrightarrow{Q}^0)\}$,
$L_2 = \{(i, i+1) | p_{i+1}(\overrightarrow{Q}^0) < p_i(\overrightarrow{Q}^0)\}$

Theorem 6

The market meets the FSIC iff $\forall l = (i, i+1) \in L_1$
$p_{i+1}(\overrightarrow{Q}^0_{L_1}, \overrightarrow{Q}^\infty_{L \setminus L_1}) > p_i(\overrightarrow{Q}^0_{L_1}, \overrightarrow{Q}^\infty_{L \setminus L_1})$ *and* $\forall l = (i, i+1) \in L_2$
$p_{i+1}(\overrightarrow{Q}^0_{L_2}, \overrightarrow{Q}^\infty_{L \setminus L_2}) < p_i(\overrightarrow{Q}^0_{L_2}, \overrightarrow{Q}^\infty_{L \setminus L_1})$

Theorem 7

If a chain-type market meets the FSIC then $\forall l \in L_1$
$M_1(l) = L_1 \setminus l, M_2(l) = L_2, \forall l \in L_2$ $M_1(l) = L_2 \setminus l, M_2(l) = L_1$. *So all edges in* L_1 *are complementary to each other and competitive to edges in* L_2*, and vice versa.*

Complementary and competitive edges for tree-type markets

$\forall l_1, l_2 \; \exists!$ path $L(l_1, l_2)$ without self-intersections including l_1 and l_2, starting at $\bar{i} \in (i_1, j_1)$ and finishing on $\bar{j} \in (i_2, j_2)$. We call l_2 initially complementary (resp. competing) for l_1, if under $\overrightarrow{Q}^0$ the flows on these edges have the same (resp. opposite) orientation with respect to $L(l_1, l_2)$. Denote $L_1^0(l), L_2^0(l)$ the sets of initially complementary and competing edges for l.

Theorem 8

A tree-type market meets the FSIC iff $\forall l = (i, j)$

$$sign(p_i(\overrightarrow{Q}^0) - p_j(\overrightarrow{Q}^0)) =$$

$$sign(p_i(\overrightarrow{Q}^0_{L_1^0(l)}, \overrightarrow{Q}^\infty_{L_2^0(l)}) - p_j(\overrightarrow{Q}^0_{L_1^0(l)}, \overrightarrow{Q}^\infty_{L_2^0(l)})).$$

Complementary and competitive edges for tree-type markets

Theorem 9

If a tree-type market meets the FSIC then $\forall l$ $M_1(l) = L_1^0(l), M_2(l) = L_2^0(l).$

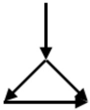

Figure 4: An example where the FSIC does not provide a possibility to determine competitive and complementary edges

Computational experiment

Cyganov (2016) considers chain-type markets with m edges, demand functions $D_i(p) = max\{0, d_i^f - c_i/2 * p\}$ and supply functions

$$S_i(p) = \begin{cases} c_i/2 * p & p \ \le \ 2d_i^f/c_i, \\ -d_i^f + c_i * p & p \ > \ 2d_i^f/c_i \end{cases}.$$

Thus, the net supply is a linar function: $\Delta S_i(p) = -d_i^f + c_i * p$. The coefficients c_i, d_i^f were chose randomly, but meet conditions $p_{i+1}^0 > p_i^0, i = 1, .., m$. For every edge $k = 1, 2, .., m$, the variable cost of transmission capacity increment ΔQ is $e_k \Delta Q^2$. Coefficients e_k are random as well as unit transportation costs e_k^t, fixed costs e_k^f and initial transmission capacities $Q_k^0, k = 1, .., m$. The next Table, Fig. 5-8 show the average and the maximal numbers of variants for the set $\overline{L}$ of expanded lines examined for determination of the optimal set $\overline{L}^*$ for every number of edges $m = 2, .., 50$.

Computational experiment

Number of edges	Number of randomly generated problems	Average number of examined variants	Maximal number of examined variants
2	1000	3,64	4
3	1000	5,60	8
4	1000	7,85	16
5	1000	10,29	32
6	1000	13,24	26
7	1000	16,14	40
8	1000	19,30	68
9	1000	23,37	258
10	1000	26,99	97
11	1000	31,28	260
12	1000	35,22	142
13	1000	41,07	167
14	1000	46,65	287
15	1000	54,95	532
16	1000	58,96	1180
17	1000	65,56	598
18	1000	76,65	1064
19	1000	86,90	1088
20	1000	96,65	1291
21	1000	109,36	1994
22	1000	121,41	2533
23	1000	188,04	32781
24	1000	159,88	8249
25	1000	175,38	8249
26	250	186,64	2129
27	250	212,87	4198
28	250	241,63	4474

Table 1:

Computational experiment

29	250	236,48	2106
30	250	658,53	65607
31	250	280,02	4168
32	250	437,92	9303
33	250	528,10	11223
34	250	548,84	26473
35	250	599,87	15987
36	250	767,60	31406
37	250	2894,81	538888
38	250	4316,50	262422
39	250	4066,20	561414
40	250	1027,83	25230
41	25	1631,52	19227
42	25	3011,44	61671
43	25	1269,80	16754
44	25	1592,72	19317
45	25	3318,48	31156
46	25	1971,52	9559
47	25	9958,08	140101
48	25	3076,48	24588
49	25	1130,20	4335
50	25	3921,68	49546

Table 2:

Computational experiment

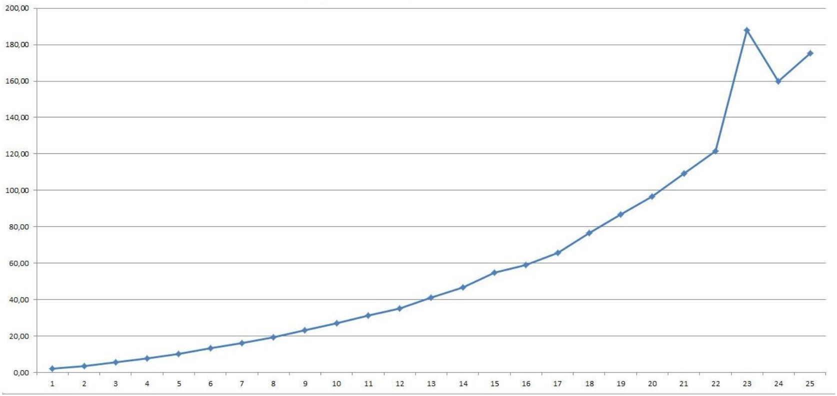

Figure 5: The average number of auxiliary problems for chains with 2-25 ages.

Computational experiment

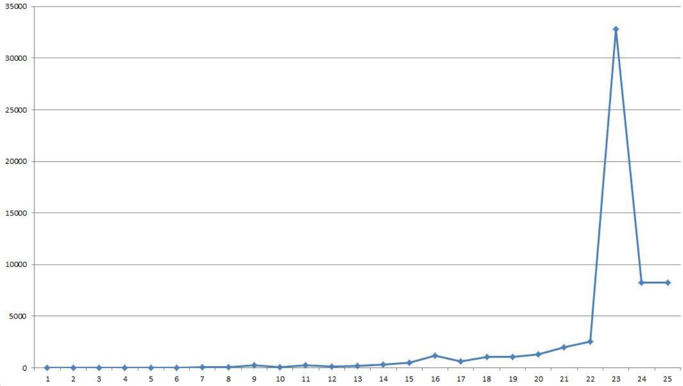

Figure 6: The maximal number of auxiliary problems for chains with 2-25 ages.

Computational experiment

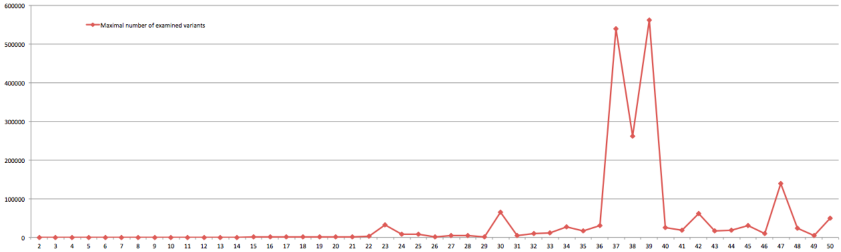

Figure 7: The maximal number of auxiliary problems for chains with 2-50 ages.

Computational experiment

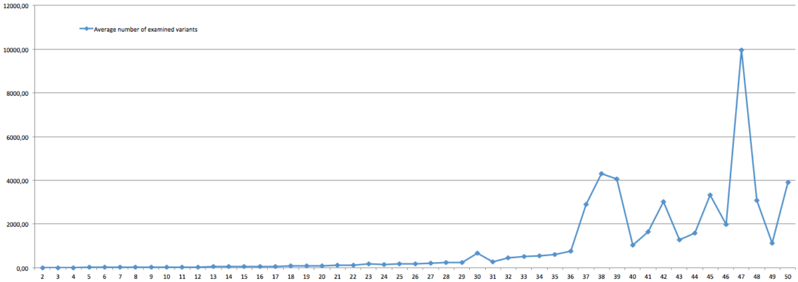

Figure 8: The average number of auxiliary problems for chains with 2-50 ages.

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